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Flat Surface Heat-Transfer Correlations for Martian Entry

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Analytic expressions have been derived for estimating the nonablating laminar and turbulent boundary-layer convective heating rates on inclined flat surfaces for the Martian atmosphere in thermochemical equilibrium. The equations are valid in the speed and altitude regime where aerobraking would occur at Mars. Comparisons with limited experimental measurements and calculations for CO_2 (the Martian atmosphere is 95.6% CO_2) yielded reasonably good agreement, especially for the ratios of heating rates in CO_2 to those in air at the same conditions. In the aerobraking speed regime, the laminar flat surface boundary layer heating rates are 15–25% greater at Mars than in air. The differences between the turbulent heating rates are even more pronounced. The turbulent heating rates can be over 50% greater at Mars than in air at the same flight conditions.

Nomenclature

c_f	= skin-friction coefficient
h	= enthalpy, J/kg
h_0	= total enthalpy at boundary-layer edge, J/kg
m.wt.	= molecular weight of gas
p_e	= pressure at boundary-layer edge, N/m^2
Pr	= Prandtl number
$\dot{q}$	= heat-transfer rate into the body per unit area, W/m^2
Re	= Reynolds number
T_w	= wall temperature, K
V_e	= velocity at boundary-layer edge, m/s
V_∞	= flight velocity, m/s
x	= distance along body surface, m
δ	= surface inclination angle with respect to freestream, deg
μ	= coefficient of viscosity, kg/m-s
ξ	= heat-transfer correlation parameter
ρ	= density of gas, kg/m^3

Subscripts

aw	= adiabatic wall condition
bt	= beginning of boundary-layer transition
c	= cone
e	= boundary-layer edge
l	= laminar boundary layer
s	= conditions behind shock
t	= turbulent boundary layer
w	= wall condition
∞	= freestream or flight conditions

Superscript

*	= evaluated at condition corresponding to reference enthalpy

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Introduction

BECAUSE of its relative similarity to Earth, Mars is the only other planet in the solar system on which men will land someday. Before such a landing, Mars will be extensively explored by unmanned vehicles. All of these vehicles will use some form of atmospheric braking, either to affect capture into planetary orbits, to descend to the surface from orbit, or to make direct entries and land on the surface.

During atmospheric passage, the vehicles are subjected to aerodynamic heating. The severity of the heating is strongly dependent on the flight velocity and altitude. In addition, the heating depends on the composition of the ambient atmosphere, which is 95.6% CO_2, 2.7% N_2, and 1.6% Ar for Mars.[1] Expressions for the stagnation point, high-speed, convective heating in pure CO_2 were derived and compared with experimental data previously.[2] More recently, numerical solution procedures using the Navier-Stokes equations[3,4] and the boundary-layer equations[5] have been developed for the chemically reacting flow of the Martian atmospheric gases. However, the numerical solutions require large amounts of computer time and memory and are not suitable for parametric systems studies or conceptual design evaluations. For such early studies, it is desirable to use relatively simple, preferably analytic, expressions to estimate the heating rates rapidly.

The objective here is to develop approximate expressions for the convective heating of flat-plate surfaces with laminar or turbulent boundary layers for the Martian atmosphere. Thermochemical equilibrium is assumed, and surface ablation is not considered. Simple transformations that relate the convective heating rates on flat plates and on cones at zero angle of attack will be given. The analytic expressions that are developed can be used in conjunction with the stagnation point equations[2] to provide approximate heating rates on the windward centerline of bodies or at other locations where the boundary layer is attached, pressure gradients are small, and no crossflow occurs.

Description of Method

Although exact numerical solutions can be found for laminar boundary layers, the equations for the turbulent boundary layer can be formulated only with the aid of experimental data such as mixing lengths, etc. Therefore, all turbulent "theories" are semiempirical. A number of approximate methods have been widely used to calculate the turbulent boundary-layer convective heating on a flat plate. All of the methods rely on Reynolds analogy that relates skin friction and heat transfer and use some form of coordinate transformation for extending the well-verified incompressible skin-friction and heat-transfer formulas to the compressible, high-speed flight conditions. Among the most widely used approaches are the reference temperature and reference enthalpy methods.

The reference temperature concept was originated by Rubesin and Johnson,[6] and the coefficients were modified by Sommer and Short.[7] The basic assumption is that a temperature within the boundary layer can be calculated, using an expression with empirically determined coefficients, which will yield the correct skin friction in a compressible, high-speed boundary layer when the incompressible skin-friction relations are used. Although the method was first formulated for laminar boundary layers, it has also been widely used for turbulent boundary layers. Eckert and Drake[8] extended the method to high-speed flows in equilibrium having real gas effects by using enthalpy in place of temperature. The most commonly used expression for the reference value of the enthalpy is[8,9]

$$h^* = 0.22h_{aw} + 0.28h_e + 0.5h_w \tag{1}$$

By using Colburn's modification of Reynolds analogy, the heat transfer to a flat plate can be written as[10]

$$\dot{q} = (c_f/2)\rho^* V_e (Pr^*)^{-0.667}(h_{aw} - h_w) \tag{2}$$

For the laminar boundary layer, the Blasius incompressible skin-friction relation is used in the form

$$\frac{c_f}{2} = \frac{0.332}{(Re^*)^{0.5}} \tag{3}$$

and the adiabatic wall enthalpy is calculated from

$$h_{aw} = h_e + 0.5(Pr_e)^{0.5} V_e^2 \tag{4}$$

For the turbulent boundary layer, Blasius' turbulent skin-friction expression (valid for $Re^* < 10^7$) is used:

$$\frac{c_f}{2} = \frac{0.0296}{(Re^*)^{0.2}} \tag{5}$$

and also

$$h_{aw} = h_e + 0.5(Pr_e)^{0.333} V_e^2 \tag{6}$$

Next, heat-transfer correlation parameters ξ are defined following the procedure of Ref. 10. The parameters are written for laminar and turbulent heat transfer, respectively, as

$$\xi_l = \frac{\dot{q}_l x^{0.5}(T_w/1500)^m}{p_e^{0.5}(V_e/V_\infty)^n (0.82 - h_w/h_0)} \tag{7}$$

and

$$\xi_t = \frac{\dot{q}_t (x - x_{bt})^{0.2}(T_w/1500)^m}{p_e^{0.8}(V_e/V_\infty)^n (0.88 - h_w/h_0)} \tag{8}$$

In Eq. (8), the expression $(x - x_{bt})$ defines the distance along the surface measured from the location where boundary-layer transition begins. The values of 0.82 in Eq. (7) and 0.88 in Eq. (8) are approximate, average Prandtl numbers raised to the exponents shown in Eqs. (4) and (6). By using the approximate values in Eq. (7) and Eq. (8), the Prandtl number is explicitly removed from the heat-transfer correlation. The heating rates are found by determining the dependence of ξ (primarily) on the flight velocity, as will be shown in the next section.

Equilibrium thermodynamic and transport properties for the Martian atmospheric gas mixture have been calculated with the codes described in Refs. 11 and 12, respectively. Tables of thermodynamic and transport properties were generated and used in the evaluation of Eqs. (1-8) for a broad range of conditions. (To be consistent with the equations given earlier, the calculated enthalpies[11] were shifted by the enthalpy of formation of CO_2, which is 393.15 kJ/mole. Also, an additional shift of 9.36 kJ/mole was made to yield zero enthalpy at 0 K instead of 298.15 K.) The flight conditions that were chosen covered velocities of 3000–10,000 m/s and ambient densities from 10^{-4} kg/m^3 to 10^{-3} kg/m^3, corresponding to altitudes of over 50 to 30 km. The surface inclination angles have been varied from 20 to 50 deg. A surface that is fully catalytic to atomic and molecular recombination has been assumed for the laminar boundary-layer cases. Turbulent boundary-layer heat-transfer measurements made in air show no significant heating reductions due to finite-rate wall catalysis. (Experimental measurements are needed to determine wall catalysis levels for laminar and turbulent boundary layers in CO_2.)

Heating-Rate Expressions

The laminar and turbulent heat-transfer correlation parameters are evaluated for the conditions listed in the preceding paragraph. Wall temperatures in the range of 1500–2000 K have been used, and it has been assumed that the dependence of ξ on T_w is sufficiently weak to set $m = 0$ in Eqs. (7) and (8). (The validity of assuming $m = 0$ is subsequently checked and will be briefly discussed later.) The exponent n of the velocity ratio has been systematically varied to approximately minimize the dependence of ξ on surface inclination angle. As expected, based on the results of Ref. 10, ξ is a strong function of flight velocity. The values of the laminar and turbulent heating correlation parameters that have been

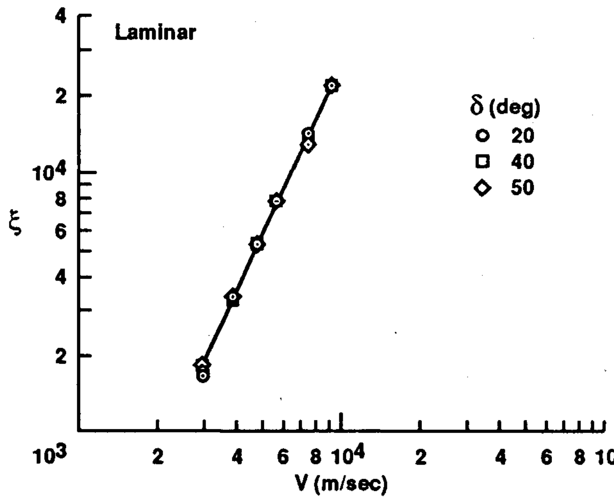

Fig. 1 Laminar heat-transfer correlation parameter.

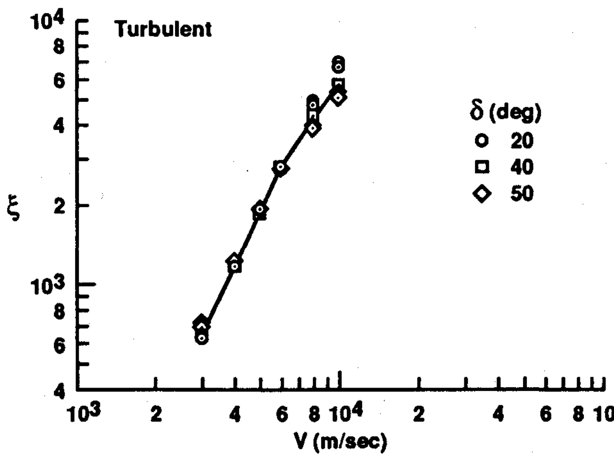

Fig. 2 Turbulent heat-transfer correlation parameter.

numerically calculated from Eqs. (7) and (8), respectively, are plotted as a function of flight velocity in Figs. 1 and 2. For the laminar case $n = 0.5$ is used, and for the turbulent case $n = 1$ is used. Although the correlation parameters have been calculated for five values of the ambient density, only the largest and smallest values of ξ are plotted in Figs. 1 and 2 for each plate angle and flight velocity. Straight lines have been fitted through the values plotted in Figs. 1 and 2, and analytic expressions are found for the lines. For example, for laminar flow

$$\xi_l = f(V_\infty) = 1.58 \times 10^{-4} V_\infty^{2.03} \tag{9}$$

and combining Eqs. (7) and (9) yields the expression for the laminar heat transfer to a fully catalytic surface, in W/m^2, for $3000 < V_\infty \leq 10{,}000$ m/s,

$$\dot{q}_l = 1.58 \times 10^{-4} (p_e V_e / V_\infty x)^{0.5} V_\infty^{2.03} (0.82 - h_w/h_0) \tag{10}$$

By the same procedure, the turbulent heat-transfer expressions are found to be, for $3000 < V_\infty \leq 6000$ m/s,

$$\xi_{t_1} = 2.70 \times 10^{-5} V_\infty^{2.12}$$

$$\dot{q}_{t_1} = 2.70 \times 10^{-5} (x - x_{bt})^{-0.2} p_e^{0.8} (V_e/V_\infty) V_\infty^{2.12} \times (0.88 - h_w/h_0) \tag{11a}$$

and for $6000 < V_\infty \leq 10{,}000$ m/s,

$$\xi_{t_2} = 1.547 \times 10^{-2} V_\infty^{1.39}$$

$$\dot{q}_{t_2} = 1.547 \times 10^{-2} (x - x_{bt})^{-0.2} p_e^{0.8} (V_e/V_\infty) V_\infty^{1.39} \times (0.88 - h_w/h_0) \tag{11b}$$

Note that in Fig. 1 the laminar calculations could be fitted by Eq. (9) to within about $\pm 6\%$. For the turbulent case (Fig. 2) the variation is larger, being a maximum of about $\pm 7\%$ for $V_\infty \leq 6000$ m/s, and reaching $\pm 15\%$ at 10,000 m/s.

For a sharp flat plate at an angle δ that is greater than about 15 deg, Eqs. (10) and (11) can be further simplified, although with a loss of a small amount of accuracy, by using the Newtonian flow approximations for pressure and velocity in the form

$$\begin{aligned} p_e &\approx \rho_\infty V_\infty^2 \sin^2\delta \\ V_e/V_\infty &\approx \cos\delta \end{aligned} \tag{12}$$

Note that by substituting Eq. (12) into Eq. (10) or (11), the expressions for the heat transfer become functions of the flight conditions and body geometry. For laminar flow, Eq. (10) becomes

$$\dot{q}_l \approx 1.58 \times 10^{-4} (\rho_\infty \cos\delta / x)^{0.5} \sin\delta \, V_\infty^{3.03} (0.82 - h_w/h_0) \tag{13}$$

and for turbulent flow, Eqs. (11) yield, for $3000 < V_\infty \leq 6000$ m/s,

$$\dot{q}_{t_1} \approx 2.70 \times 10^{-5} (x - x_{bt})^{-0.2} (\rho_\infty \sin^2\delta)^{0.8} \cos\delta \, V_\infty^{3.72} \times (0.88 - h_w/h_0) \tag{14a}$$

and for $6000 < V_\infty \leq 10{,}000$ m/s,

$$\dot{q}_{t_2} \approx 1.547 \times 10^{-2} (x - x_{bt})^{-0.2} (\rho_\infty \sin^2\delta)^{0.8} \cos\delta \, V_\infty^{2.99} \times (0.88 - h_w/h_0) \tag{14b}$$

The inclined flat-plate heating-rate expressions given by Eqs. (10), (11), (13), and (14) can also be applied to pointed circular cones at zero angle of attack. For the laminar boundary layer, the Mangler transformation applies and yields for the heating rate on a cone, with a half-angle that is equal to the flat plate's inclination angle,

$$\dot{q}_{c_l} = 3^{0.5} \dot{q}_l \tag{15}$$

The transformation, which is derived in a general form in Ref. 9, gives for the turbulent boundary layer approximately

$$\dot{q}_{c_t} = 1.18 \dot{q}_t \tag{16}$$

Note that the differences in the heating of the two geometric shapes is much greater for a laminar boundary layer than for a turbulent one. The Newtonian flow approximations, Eq. (12), also apply to sharp cones at zero angle of attack; in fact, Eq. (12) is more accurate for sharp cones than for inclined flat plates.

A limited number of calculations have been made to determine the dependence of the correlation parameter on the wall temperature directly. For wall temperatures from 1000 to at least 2000 K, the laminar heat-transfer parameter varies approximately as

$$\dot{q}_l \sim (T_w/1500)^{-0.06} \tag{17}$$

For the turbulent cases, the heating rate can vary somewhat more with wall temperature; the exponent in Eq. (17) can approach -0.1. However, it is questionable whether the inclusion, in an approximate formulation, of such minor effects is meaningful. The assumption that $m = 0$ is justified, especially in light of the comparisons that will be presented in the next section.

Comparison with Other Results

It is essential to compare the approximate expressions that have been derived here with experiments and other calculations to lend some credence to the present results. Unfortunately, there is very little high-speed, experimental, heat-transfer data available for the CO_2 laminar boundary layer, apart from the stagnation point measurements that are summarized in Refs. 2 and 13. Futhermore, no high-speed heat-transfer measurements could be found that were made in CO_2 flows having turbulent boundary layers. Therefore, results from turbulent boundary-layer expressions derived here will be compared with values that have been computed using other methods. In addition, the ratio of the heat transfer in the Martian gases to that in air will be compared for both the laminar and turbulent flat-plate boundary layers.

In 1970, Stewart[14] measured the forebody heat transfer on models of the Viking Mars entry vehicle both in air and in CO_2. The laminar heating-rate comparison is shown in Fig. 3. (The Viking vehicle's forebody was a 70-deg half-angle cone with a nose radius that was equal to one-half the body's radius.) The tests were conducted in the Ames 1.07 m (42 in.) combustion-driven shock tunnel at a test-section stream velocity of approximately 4250 m/s. The test stream Reynolds number, based on model diameter, was about 7×10^3. Metallic models and thermocouples were used; therefore, the surfaces were fully catalytic. The only information that could be gleaned from Fig. 3 is the ratio of the heat transfer in CO_2 to that in air. The ratio was determined at 12 forebody locations and varied from 1.25 to 1.39. This range of values is plotted in Fig. 4 and compared with the ratio of calculated Martian gas flat-plate, laminar heat transfer to that for air and gives a value of 1.29. The Martian gas laminar heating rates were calculated using Eq. (13) and were normalized by using an equivalent laminar heating expression for air.[15] Although the blunted cone measurements are compared with flat-plate calculations in Fig. 4, the agreement is good, very possibly because normalized values are compared. Also shown in Fig. 4, but at 8 km/s, is a narrow range of heating rate ratios that

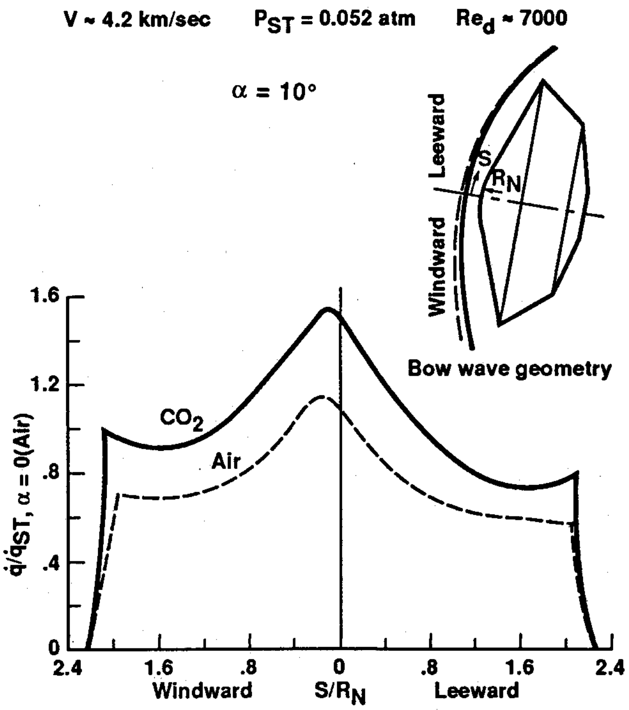

Fig. 3 Effect of gas composition on measured heating rates on Viking forebody.[14]

were computed with the boundary-layer equations for both air and CO_2 using the BLIMP code.[16,17] The heating rates in both gases were calculated[18] at the windward centerline of a blunt-nosed biconic at a 20-deg angle of attack. The vehicle was flying at 8 km/s, and the ambient gas density was 1.33×10^{-4} kg/m^3. The calculated heating-rate ratios[18] on the aft frustum of the biconic vehicle range from 1.12 to 1.15, compared with a value of 1.16 for the present correlation. (The slight difference in composition between the Martian gas and pure CO_2 is expected to have a negligibly small effect on the convective heating. Because the geometry of the body that was studied in Ref. 18 differed greatly from the flat surfaces considered here, comparisons of absolute heating rates were not made.) However, the agreement confirms the conclusion that the laminar heating rates in the Martian gas can be from 12 to 40% higher than in air, over the speed range that is covered in Fig. 4. Furthermore, the approximate relationship between the laminar heating rates that has been derived here in both gases (the line in Fig. 4) can be used to quickly estimate the heating rates on a flat surface in the Martian atmosphere when the heating has previously been calculated in air. The expression has the form

$$(\dot{q}_{CO_2}/\dot{q}_{air})_l \approx 5.363 V_\infty^{-0.17} \tag{18}$$

and comes from using Eq. (13) with the corresponding equation for air.[15]

Since no high-speed, turbulent heat-transfer measurements are known for predominantly CO_2 flows, comparisons will be made with two other calculations. The first one is a turbulent flow computation that was done using a semiempirical method developed by Spalding and Chi,[19] and the second used the BLIMP code.[18] Calculations were made in CO_2 and in air using both methods. The computation using the BLIMP code was performed on the same biconic body and at the same flight condition that was just described for the laminar flow heating rates; however, an early boundary-layer transition criterion was imposed.[18]

The values from the Spalding and Chi method have been calculated by Savage.[20] The following conditions were used for the four test cases: $V_\infty = 5000$ and 7000 m/s, $\rho_\infty = 7 \times 10^{-4}$ kg/m^3, $x - x_{bt} = 10$ m, and $T_w = 1500$ K. The results are presented in Table 1. For the reference enthalpy h^* values that have been calculated here, Eq. (13) is used for the Martian gas. For consistency, a corresponding turbulent heating correlation equation for air was developed and is given in Appendix A. Equation (A1) differs somewhat from the expressions derived in Ref. 10, partly because more accurately calculated transport properties[21] have been used here.

As can be seen from Table. 1, the present h^* method predicts heating rates that are generally higher. The average and maximum differences between the present method and that of Spalding and Chi are 19 and 26% for the Martian gas and CO_2 and 18 and 30% for air. The differences for the Martian gas and CO_2 are within the range of uncertainty of either method. The relative ability of the reference enthalpy method and Spalding and Chi's formulation to predict turbulent heating in air is briefly discussed next.

It has been documented[22,23] previously that, when comparisons with data measured in air were made, the Spalding and Chi method can underpredict the turbulent heating rates significantly. However, another comparison[24] found reasonably good agreement between the data and the Spalding and Chi method. It is also well documented[23,24] that the reference enthalpy methods tend to overpredict the heating rates when the ratio of the wall temperature to the total temperature is below about 0.25. For the example results presented in Table 1, $T_w/T_0 = 0.44$ for $V_\infty = 5000$ m/s, but decreases to $T_w/T_0 = 0.23$ for $V_\infty = 7000$ m/s. More fundamental is the observation[22-24] that 20–30% disagreement was common between measured turbulent heating rates and values that were calculated using the various approximate "theories," including the two compared here. Although based on very limited comparisons, the agreement of the Martian gas and the CO_2 heating rates shown in Table 1 is adequate, considering the simplicity of the expressions that have been derived.

The ratio of the turbulent heating rates for the Martian gas and in CO_2 to that in air is shown in Fig. 5. Included in Fig. 5 are the ratios of the heating rates for the Spalding and Chi method from Table 1. Although the absolute heating rates predicted by the h^* method and by Spalding and Chi can differ significantly (as shown in Table 1), the ratios of heating rates at Mars to those in air agree well. For example, at 5 km/s Spalding and Chi's heating ratios are 1.48 and 1.60 compared with the h^* value of 1.56. At 7 km/s, Spalding and Chi's ratios are 1.48 and 1.51, whereas the h^* method gives 1.47. Also shown in Fig. 5 is the range of turbulent heating rates that were calculated[18] at 8 km/s, on the aft frustum of the biconic vehicle's windward centerline. The body locations chosen were far downstream from the points where transition began, since these points were different in air and CO_2. The heating ratios found from the calculations in Ref. 18 varied from 1.30 to 1.33; by comparison, the h^* method yields a value of 1.35 (see Fig. 5). (Absolute heating-rate values were not compared, again, because the body geometry[18] was very different from a flat plate.)

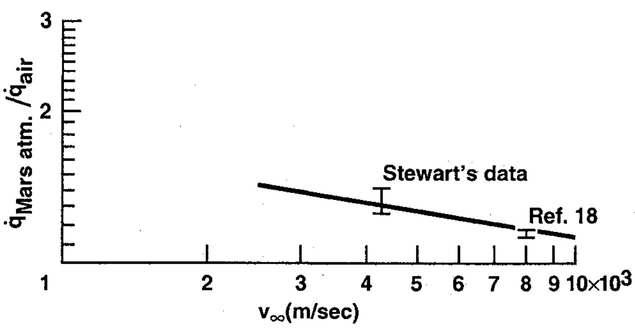

Fig. 4 Comparison of laminar heat-transfer ratio with measurements and other calculations.

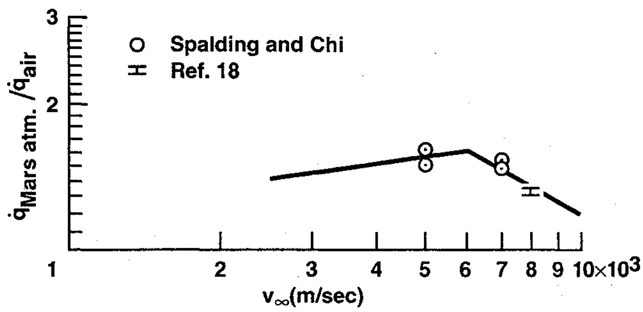

Fig. 5 Comparison of turbulent heat-transfer ratio with other calculations.

Table 1 Turbulent boundary-layer heating-rate comparisons (W/cm²)

Method and parameters	Velocity, m/s	
	5000	7000
Reference enthalpy, h^*		
$\delta = 40$ deg		
Martian atmosphere	83.1	282
Air	53.1	192
$\delta = 20$ deg		
Martian atmosphere	37.1	126
Air	23.7	86.0
Spalding and Chi		
$\delta = 40$ deg		
CO_2	73.1	245
Air	49.5	166
$\delta = 20$ deg		
CO_2	31.6	99.7
Air	19.7	66.0

The approximate expressions for the turbulent heating-rate ratios that are plotted in Fig. 5 are given next. These relations can be used to quickly estimate the turbulent heating rates in the Martian atmosphere if the heating rates are known in air. Equations (14a), (14b), and (A1) were used. The expressions are, for flight velocities of about 3000–6000 m/s

$$(\dot{q}_{CO_2}/\dot{q}_{air})_t \approx 0.473 V_\infty^{0.14} \tag{19a}$$

and for velocities of 6000–10,000 m/s

$$(\dot{q}_{CO_2}/\dot{q}_{air})_t \approx 271 V_\infty^{-0.59} \tag{19b}$$

The ratios given by Eqs. (18), (19a), and (19b) are independent of altitude, body inclination angle, and distance along the surface. (Of course, the preceding parameters are used to compute the heating in air.) Note that, in the important aerobraking flight regime from 8000 to 5000 m/s, the present correlation predicts turbulent heating rate increases, over the rates experienced in air, that range from 35 to 56%, respectively. Therefore the flat surface turbulent heating rates at Mars exceed the air values by more than the laminar heating rates over most of the flight regime that is of primary interest for aerobraking. (The stagnation point heating-rate expression in a CO_2 atmosphere[25] gives heating rates that are only 5–10% higher than in air at the same flight conditions. The reason why the stagnation point heating rate in CO_2 differs so little from the value in air, compared with the laminar flow on a flat surface, is explained briefly in Appendix B.)

Concluding Remarks

Analytic expressions have been derived for estimating the nonablating, laminar, and turbulent boundary-layer convective heating rates on inclined flat plates for the Martian atmosphere in thermochemical equilibrium. The equations are valid in the speed and altitude regime where aerobraking would occur at Mars. The laminar boundary-layer heating-rate ratios found from values measured both in CO_2 and in air compared well with results from the present formulation. Limited comparisons of the turbulent boundary-layer heating with values calculated for CO_2, using the approximate method of Spalding and Chi, indicate that the reference enthalpy approach that is used here overpredicts the heating by an average of 19%. However, differences of over 20% are not unusual among the various approximate methods that are used to predict turbulent boundary-layer heat transfer at high speeds. The ratios of heating rates in CO_2 to those in air that were calculated at the same conditions by using Spalding and Chi for turbulent heating and, in addition, a boundary-layer code agreed well for both laminar and turbulent flows. Therefore, it is recommended that, if codes are available that predict the heating rates reliably in air, the use of Eqs. (18), (19a), and (19b) should be considered for calculating the laminar and turbulent heating, respectively, in the Martian atmosphere. A major advantage of the analytic heating relations that have been presented is that they can be efficiently used for Martian mission studies where many repetitive calculations must be performed.

Appendix A

The reference enthalpy method was used to calculate the heating on inclined flat plates in air using the formulation of Ref. 10 but employing the transport properties of Ref. 21. The computed heat-transfer values were fitted with an equation having the form of Eq. (9) to a numerical accuracy of about ± 9%. The resultant turbulent heat-transfer expression, which is applicable in the speed range of 3000–13,000 m/s, is

$$\dot{q}_t = 5.70 \times 10^{-5}(x - x_{bt})^{-0.2} p_e^{0.8} \times (V_e/V_\infty) V_\infty^{1.98}(0.88 - h_w/h_0) \tag{A1}$$

If the Newtonian approximations for boundary-layer edge pressure and velocity, Eq. (12), are used, then Eq. (A1) becomes

$$\dot{q}_t = 5.70 \times 10^{-5}(x - x_{bt})^{-0.2}(\rho_\infty \sin^2\delta)^{0.8} \cos\delta V_\infty^{3.58} \times (0.88 - h_w/h_0) \tag{A2}$$

Appendix B

Approximate expressions are derived to illustrate how laminar boundary-layer heating depends on gas composition and how that dependence differs for flat surface and stagnation point flows. From Eqs. (2) and (3), it can be seen that approximately

$$\dot{q} \sim \rho^* V_e (Re^*)^{-0.5} \sim (\rho^* V_e \mu^*/x)^{0.5} \tag{B1}$$

After substituting for the density using the thermal equation of state, Eq. (B1) becomes

$$\dot{q} \sim [p(\text{m.wt.})^* \mu^* V_e/(\mathcal{R} T^* x)]^{0.5} \tag{B2}$$

where $\mathcal{R}$ is the universal gas constant. For the laminar heating on a flat surface, Eq. (B2) gives, to first order,

$$\dot{q} \sim [(\text{m.wt.})^* \mu^*]^{0.5} \tag{B3}$$

Since the coefficients of viscosity in the Martian atmosphere (m.wt. = 43.5), CO_2 (m.wt. = 44), and air do not differ greatly,[26] the heating-rate ratio depends on the molecular weights of the gases, to first order, and is about 1.23. (Note that 1.23 is the approximate average value of the range shown in Fig. 4.)

At the stagnation point, V_e is undefined but can be written as

$$\frac{V_e}{x} \approx \frac{dV_e}{dx} \tag{B4}$$

The velocity gradient can be found from Newtonian theory[9,15] for a spherical segment and is

$$\left(\frac{\mathrm{d}V_e}{\mathrm{d}x}\right)_{x=0} \approx \left(\frac{V_\infty}{r_n}\right)\left(\frac{2\rho_\infty}{\rho_s}\right)^{0.5} \tag{B5}$$

where r_n is the nose radius. Therefore, when Eq. (B2) is applied at the stagnation point, it becomes

$$\dot{q} \sim [(\mathrm{m.wt.})^*\mu^*]^{0.5}(\rho_\infty/\rho_s)^{0.25} \tag{B6}$$

The major distinction between Eqs. (B3) and (B6) is the presence of the density ratio term in the latter. The density ratio is substantially different in CO_2 and air. For example, at aerobraking atmospheric densities and at speeds of 5 and 8 km/s, the density ratios across the normal shock wave on the stagnation streamline are, approximately, 0.043 and 0.046 for CO_2,[27] and 0.087 and 0.062 for air,[28] respectively. Therefore, the stagnation point heating rate ratios become, approximately, 1.03 and 1.14 at 5 and 8 km/s, respectively, and are substantially lower than the flat surface value of 1.23.

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